

Isochron Dating

We can rearrange the radioactive decay equation in two more ways:

$$[X]_t = [X]_i e^{-kt}$$

$$[X]_t = [X]_i \left(\frac{1}{2}\right)^{\left(\frac{t}{t_{1/2}}\right)}$$

Since it is easier to measure concentration than it is to count atoms, we have expressed the values in terms of molar concentrations $[X]$ instead of number of atoms N . The problem is that we rarely know the initial concentration $[X]_i$ offhand. However, we can often figure it out by measuring the amount of decay *product* that accumulates.

1. Suppose we have a mineral sample that contains a concentration $[R]_i$ of some specific radioactive isotope *when the mineral formed*. Write an equation to express the concentration $[R]_t$ after time t has elapsed?

2. Radioactive atoms don't disappear completely when they decay; they turn into other atoms, of some "daughter" element isotope. What is the concentration $[D]_t$ of this daughter isotope at time t , if the sample had some initial concentration of $[D]_i$ to begin with at $t=0$ (i.e., when the mineral formed)?

3. Suppose a given sample has (at its formation) $[R]_i = 1\%$ and $[D]_i = 0.5\%$; and the half-life of the radioactive element R (as measured in a laboratory) is known to be 1 billion years. Use the table below to figure out how $[R]$ and $[D]$ change with time.

Time:	0	1 billion years	2 billion years	3 billion years	4 billion years	5 billion years
$[R]$	1.0%					
$[D]$	0.5%					

4. Now plot these points on a graph, labelling the points with their ages.

Put $[R]$ on the X axis and $[D]$ on the Y axis. Each axis should go from 0% to 2.5%

5. Suppose you pick up a rock in the field and measure $[R]$ and $[D]$ (their values today), and you find that in one mineral in the rock, $[R] = 0.5\%$ and $[D] = 1\%$. Can you figure out its age?

6. Suppose now that two other mineral samples from the rock had different initial concentrations of $[R]_i$. How do $[R]$ and $[D]$ change with time now?

Sample 2

Time:	0	1 billion years	2 billion years	3 billion years	4 billion years	5 billion years
$[R]$	1.5%					
$[D]$	0.5%					

Sample 3

Time:	0	1 billion years	2 billion years	3 billion years	4 billion years	5 billion years
$[R]$	2.0%					
$[D]$	0.5%					

7. Plot this new data on the same graph. Using a ruler draw a line through each set of points with the same age and extrapolate to the Y axis. What information does this give you? The lines that you drew are called "isochrons".

8. Suppose we find a rock, and measure the concentrations [R] and [D] in a bunch of different mineral grains within the rock:

	[R]	[D]	[D _{iso}]
Mineral A	1.0%	4.0%	1.0%
Mineral B	0.1%	1.5%	1.0%
Mineral C	0.5%	2.5%	1.0%
Mineral D	0.4%	2.2%	1.0%
Mineral E	0.4%	1.7%	0.5%

Plot these on a separate graph (this time the Y axis must go to 4).
(Ignore the D_{iso} column for now.)

9. What was the initial concentration [D]_i in this rock? Use the technique from (7), and ignore mineral E for now.

10. Now that you know [D]_i, draw an isochron for t=0, when we assume [D] was constant everywhere.

11. Draw an isochron for after 1 half-life has elapsed (t = 1 billion years), and after 2 half-lives (2 billion years). How old is the rock?

12. What would happen to the t=0 isochron line if [D]_i were not a constant – that is, different regions of the rock had different values of [D] as well as of [R]? What about at other times? Mineral E doesn't fit the pattern. You might suspect (and you'd be right) that this has something to do with its different value of [D_{iso}], which refers to the concentration of some alternate isotope of the same element as the daughter product. As it turns out, there is no reason to expect that the concentration of any isotope should be the same in different regions of the sample – this is why we are able to get different measurements for [R] in different places, and we expect this to happen as well [D]: there is no single initial value [D]_i as we imagined in (9).

Fortunately, we can correct for this, knowing the abundance of some other isotope of the daughter element, [D_{iso}]. As it turns out, if some region of a rock happens to be depleted in one isotope of some element, it will be depleted in *all* isotopes of that element by the same fraction (since different isotopes of an element are chemically identical for most purposes). So we can *divide* all our concentrations by this alternate isotope fraction [D_{iso}] to convert everything to a unit system in which the points really do begin on a line, since [D]/[D_{iso}] is constant.

We know the value of [D_{iso}] as long as it is not the daughter product of some other radioactive decay process. In such a case, [D_{iso}] is constant and was the same at all times in the past as now.

13. "Correct" for the fact that mineral E was depleted in the daughter element to begin with by dividing its values (both [R] and [D]) by [D_{iso}]. Does it fit the pattern now?